Appendix C: A Lapse-Rate Tendency Equation based on Equivalent Potential Temperature and Potential Instability

We previously developed a lapse-rate tendency equation quantifying how potential temperature varies with height. This directly connects to an *air parcel's conditional stability*, where saturation (or the lack thereof) is the relevant stability criterion. However, it is also possible to develop a lapse-rate tendency equation that quantifies how equivalent potential temperature (which includes both temperature and moisture) varies with height. This directly connects to a *layer's potential stability*, which describes how a layer's stability changes as a result of changes in its altitude. Here, we introduce and interpret a lapse-rate tendency equation where the lapse rate is defined by equivalent potential temperature, where:

$$\Gamma_m = -\frac{\partial \theta_e}{\partial p} \quad \text{(C1)}$$

$\overline{\Gamma_m}$ beneath the inversion is negative (equivalent potential temperature decreasing with height, a potentially unstable situation) whereas it is positive above the inversion (equivalent potential temperature increasing with height, a potentially stable situation).

A lapse-rate tendency equation for the quantity in (C1) is given by:

$$\frac{\partial \overline{\Gamma_m}}{\partial t} = -\frac{\partial \overline{\vec{v}_h}}{\partial p} \cdot \nabla \overline{\theta_e} - \overline{\vec{v}} \cdot \nabla \overline{\Gamma_m} - \overline{\omega} \frac{\partial \overline{\Gamma_m}}{\partial p} - \overline{\Gamma_m} \frac{\partial \overline{\omega}}{\partial p} + \frac{\partial^2}{\partial p^2}\left(\overline{\omega' \theta_e'}\right) - \frac{\partial}{\partial p}\left[\frac{1}{c_p}\left(\frac{p_o}{p}\right)^{\frac{R}{c_P}} \sum_i \overline{H_i}\right] \quad \text{(C2)}$$

If we again neglect precipitation processes, H_i is composed of surface evaporation, radiative processes, and sensible heating.

Our stability criteria are $\frac{\partial \overline{\Gamma_m}}{\partial t} < 0$ for destabilization (a more-rapid decrease of θ_e with increasing height is associated with greater potential instability) and $\frac{\partial \overline{\Gamma_m}}{\partial t} > 0$ for stabilization (a less-rapid decrease of θ_e with increasing height is associated with greater potential stability).

We now consider how each of the terms in (C2) modify the equivalent potential temperature lapse rate.

(a) Differential horizontal equivalent potential temperature advection

This term has the same physical interpretation as its potential-temperature equivalent. Given small large-scale horizontal $\overline{\theta_e}$ gradient magnitudes and weak large-scale vertical wind shear across the trade-wind inversion, this term is negligibly small.

(b) Horizontal lapse-rate advection

This term has the same physical basis as its potential-temperature equivalent, wherein advection can only move an inversion and cannot create or destroy an inversion.

θ_e below the inversion decreases most rapidly with increasing height in eastern subtropical oceans, with less-rapid decreases with increasing height further to the west. Since the large-scale-averaged horizontal flow is directed from east to west across this region, this term causes θ_e to decrease more rapidly with height beneath the inversion, a destabilizing scenario (if potential instability is realized through lifting the below-inversion layer upward).

(c) Vertical lapse-rate advection

This term also has the same physical basis as its potential-temperature equivalent, where advection can only move (and not create or destroy) an inversion. Large-scale descent through the inversion can lower its altitude but otherwise not affect its intensity.

(d) Divergence

Because $\overline{\Gamma_m}$ beneath the inversion is negative, large-scale divergence acts as a destabilizing term in this framework. The magnitude of this influence is small, however, compared to the dry case.

(e) Turbulent vertical heat flux

Equivalent potential temperature is at a minimum at inversion base and increases above and below the inversion. Thus, cloud-scale ascent and descent will mix higher θ_e into the inversion, such that θ_e increases where it is smallest. This causes $\overline{\Gamma_m}$ in the boundary layer to become less negative, and thus this term is a stabilizing term. This term is the largest contributor to stability in this framework.

(f) Heating terms

Surface Evaporation: Evaporation of moisture into the air from the ocean moistens the near-surface layer. This raises θ_e at the surface, destabilizing the boundary layer. Therefore, surface evaporation is a destabilizing term.

Sensible Heating: As before, in the eastern subtropical oceans, sea-surface temperature is less than the near-surface air temperature. The sensible cooling of the near-surface layer that results reduces θ_e and thus stabilizes the boundary layer. Conversely, sea-surface temperature is greater than the near-surface air temperature in the central subtropical oceans. The sensible warming of the near-surface layer that results increases θ_e and thus destabilizes the boundary layer.

Radiative Processes: As before, radiative processes cool near the inversion's top and warm beneath the inversion. The mathematical and physical interpretations are thus the same as for the dry case, such that radiation destabilizes the boundary layer.

Thus, turbulent vertical mixing in clouds and sensible heating (in the eastern oceans) stabilize the boundary layer beneath the trade-wind inversion. Divergence, surface evaporation, radiative processes, and sensible heating (in the central oceans) destabilize the boundary layer beneath the trade-wind inversion. Advection can transport enhanced (vertical motions) or reduced (horizontal motions) instability in the boundary layer.