Trade-Wind Inversion

Introduction

The Hadley cell's descending branch facilitates the formation and maintenance of a temperature inversion, which separates the atmospheric boundary layer from the free atmosphere above, known as the trade-wind inversion at subtropical latitudes. In this lecture, we first expand on this basic definition for the trade-wind inversion, following which we discuss the trade-wind inversion's climatological structure. Subsequently, we develop lapse-rate tendency equations, one for potential temperature (related to conditional instability; found in this section) and one for equivalent potential temperature (related to potential instability; found in Appendix C), to allow us to connect physical processes to the trade-wind inversion's altitude and intensity.

Key Questions

- What are the climatological structure and impacts of the trade-wind inversion?
- How does the trade-wind inversion's intensity, as measured by the rate at which temperature increases with altitude across the inversion, vary spatially?
- What physical processes cause the trade-wind inversion to form and/or change in intensity?

Trade-Wind Inversion Structure and Impacts

The trade-wind inversion is characterized by a layer in which temperature increases with increasing altitude, with this layer separating the atmospheric boundary layer from the free atmosphere. Trade-wind inversions are commonly found in association with the Hadley cell's descending branch in the subtropical oceans and adjacent coastal regions. They are typically strongest and found at lower altitudes in the eastern subtropical oceans where sea-surface temperatures are comparatively low due to equatorward-directed ocean currents east of the subtropical anticyclones. These inversions weaken and become located at progressively higher altitudes both westward and equatorward of the eastern subtropical oceans. There is modest diurnal (daily) and seasonal variability on the trade-wind inversion's structure, with stronger large-scale descent resulting in stronger inversions at lower altitudes.

The trade-wind inversion limits clouds and turbulent vertical mixing to altitudes below the inversion's base. In the eastern subtropical oceans, where the inversion is typically strongest and located at lowest altitudes, thunderstorms are infrequent (particularly during the local summer months) and precipitation is dominated by infrequent light drizzle from stratus clouds beneath the inversion. Fog and haze, resulting from relatively warm near-surface air passing over the relatively cool ocean surface beneath the inversion, commonly occur in the eastern subtropical oceans. Predominantly stratus clouds in the far eastern subtropical oceans become more cellular (stratocumulus, then cumulus) as you move westward and/or equatorward. The average cloud fraction (for all cloud types) beneath the inversion is ~60% in the eastern subtropical oceans and decreases as you move westward and/or equatorward.

Trade-Wind Inversion Formation and Modulation

By definition, trade-wind inversions are associated with temperatures that increase with height over a short distance in the lower troposphere and thus are associated with positive lapse rates. We can use the first law of thermodynamics to derive an equation, known as a lapse-rate tendency equation, to quantify the physical influences on the trade-wind inversion's altitude and strength.

The first law of thermodynamics can be expressed as:

$$c_p \frac{T}{\theta} \frac{D\theta}{Dt} = \sum_i H_i \tag{1}$$

where H_i are all heating sources and sinks, including latent heating, sensible heating, and radiation. We also find it helpful to recall Poisson's equation, relating temperature and potential temperature, given by:

$$\theta = T \left(\frac{p_o}{p}\right)^{\frac{R}{c_P}} \tag{2}$$

If we solve (2) for T and substitute into (1), we obtain:

$$\frac{D\theta}{Dt} = \frac{1}{c_p} \left(\frac{p_o}{p}\right)^{\frac{R}{c_P}} \sum_i H_i \tag{3}$$

The total derivative in (3) contains local change (t), horizontal advection (x, y), and vertical advection (here, p rather than z) terms. Expanding (3) using this, we obtain:

$$\frac{\partial \theta}{\partial t} = -\vec{v}_h \bullet \nabla_h \theta - \omega \frac{\partial \theta}{\partial p} + \frac{1}{c_p} \left(\frac{p_o}{p}\right)^{\frac{R}{c_P}} \sum_i H_i \tag{4}$$

where subscripts of h denote horizontal quantities and operators.

It can be shown that:

$$\nabla \cdot (\boldsymbol{v}\theta) = \boldsymbol{v} \cdot \nabla\theta + \theta(\nabla \cdot \boldsymbol{v})$$

In the isobaric coordinate system used here, the continuity equation states that $\nabla \cdot \boldsymbol{v} = 0$, such that:

$$\nabla \cdot (\boldsymbol{v}\theta) = \boldsymbol{v} \cdot \nabla\theta$$

Using this to rewrite (4), we obtain:

$$\frac{\partial \theta}{\partial t} = -\nabla_h \cdot (\boldsymbol{v}_h \theta) - \frac{\partial}{\partial p}(\omega\theta) + \frac{1}{c_p} \left(\frac{p_o}{p}\right)^{\frac{R}{c_P}} \sum_i H_i \tag{5}$$

Each of the variables in (5) can be written as the sum of a mean (overbar) and perturbation (prime) quantity (e.g., $u = \overline{u} + u'$) except for H_i (since $H_i' = 0$). We wish to substitute these expansions into (5), after which we wish to take the Reynolds average so that the resulting equation applies on larger (average) scales. Doing this, we obtain:

$$\frac{\partial}{\partial t}\overline{(\overline{\theta}+\theta')} = -\frac{\partial}{\partial x}\left(\overline{(\overline{u}+u')(\overline{\theta}+\theta')}\right) - \frac{\partial}{\partial y}\left(\overline{(\overline{v}+v')(\overline{\theta}+\theta')}\right) - \frac{\partial}{\partial p}\left(\overline{(\overline{\omega}+\omega')(\overline{\theta}+\theta')}\right) + \frac{1}{c_p}\left(\frac{p_o}{p}\right)^{\frac{R}{C_P}}\sum_i \overline{H_i} \tag{6}$$

We can simplify (6) using Reynolds' postulates, where for any two variables *a* and *b*, $\overline{\overline{a}} = \overline{a}$, $\overline{a'} = 0$, $\overline{a'\overline{b}} = 0$, and $\overline{a'b'} \neq 0$. Doing so, we obtain:

$$\frac{\partial\overline{\theta}}{\partial t} = -\frac{\partial}{\partial x}\left(\overline{u\theta}\right) - \frac{\partial}{\partial x}\left(\overline{u'\theta'}\right) - \frac{\partial}{\partial y}\left(\overline{v\theta}\right) - \frac{\partial}{\partial y}\left(\overline{v'\theta'}\right) - \frac{\partial}{\partial p}\left(\overline{\omega\theta}\right) - \frac{\partial}{\partial p}\left(\overline{\omega'\theta'}\right) + \frac{1}{c_p}\left(\frac{p_o}{p}\right)^{\frac{R}{C_P}}\sum_i \overline{H_i} \tag{7}$$

In (7), the second and fourth right-hand side terms represent turbulent (i.e., small-scale) horizontal mixing and are negligibly small. Invoking the relationship that $\nabla \cdot (\boldsymbol{v}\theta) = \boldsymbol{v} \cdot \nabla\theta$, we can rewrite (7) as:

$$\frac{\partial\overline{\theta}}{\partial t} = -\overline{u}\frac{\partial\overline{\theta}}{\partial x} - \overline{v}\frac{\partial\overline{\theta}}{\partial y} - \overline{\omega}\frac{\partial\overline{\theta}}{\partial p} - \frac{\partial}{\partial p}\left(\overline{\omega'\theta'}\right) + \frac{1}{c_p}\left(\frac{p_o}{p}\right)^{\frac{R}{C_P}}\sum_i \overline{H_i} \tag{8}$$

The lapse rate of potential temperature is defined as:

$$\Gamma = -\frac{\partial\theta}{\partial p} \tag{9}$$

Large positive values of (9), such as through an inversion, represent absolute stability. Small positive values of (9) typically represent conditional instability. Neutrality to vertical parcel displacements in a subsaturated environment occurs when (9) equals zero. Finally, negative values of (9) represent absolute instability in a subsaturated environment.

If we take the derivative of (8) with respect to *p*, multiply by -1, and substitute with (9), we obtain:

$$\frac{\partial\overline{\Gamma}}{\partial t} = \frac{\partial\overline{\vec{v}}}{\partial p} \cdot \nabla\overline{\theta} - \overline{\vec{v}} \cdot \nabla\overline{\Gamma} - \overline{\omega}\frac{\partial\overline{\Gamma}}{\partial p} - \overline{\Gamma}\frac{\partial\overline{\omega}}{\partial p} + \frac{\partial^2}{\partial p^2}\left(\overline{\omega'\theta'}\right) - \frac{\partial}{\partial p}\left[\frac{1}{c_p}\left(\frac{p_o}{p}\right)^{\frac{R}{C_P}}\sum_i \overline{H_i}\right] \tag{10}$$

(10) expresses the local time-rate of change in the large-scale–averaged potential-temperature lapse rate in terms of six right-hand-side forcing terms. In other words, (10) allows us to quantify how various physical processes change the lapse rate and thus influence inversion strength. $\vec{v}$ and ∇ represent the horizontal wind and horizontal gradients, respectively.

We now wish to consider how each of the six right-hand-side terms in (10) impacts the lapse rate.

(a) Differential horizontal potential-temperature advection

This term is comprised of the vertical variation in the large-scale-averaged horizontal wind dotted into the horizontal gradient of the large-scale-averaged potential temperature. This general structure is broadly consistent with that of advection (wind dotted into the gradient of a quantity). As a result, we refer to this term as *differential horizontal potential-temperature advection*, where "differential" refers to the advection being by the horizontal wind's vertical variation.

Even through the trade-wind inversion, horizontal $\overline{\theta}$ gradients have small magnitudes – on the order of 2 K per 500 km. Since the vertical wind shear of the large-scale-averaged flow typically also has small magnitude, we say that this term is negligibly small for the trade-wind inversion.

(b) Horizontal lapse-rate advection

This term is comprised of a velocity dotted into the gradient of a quantity, inherently representing advection. Specifically, this term represents horizontal advection *by the large-scale-averaged wind* of the *large-scale-averaged* lapse rate.

If there is no horizontal lapse-rate gradient, this term cannot create one – it can only move (advect) an existing gradient. Horizontal $\overline{\Gamma}$ gradients are small except in the eastern subtropical oceans. The lower-tropospheric horizontal flow in these regions is directed from areas of larger to smaller large-scale-averaged lapse rates, such that this term contributes to a stronger trade-wind inversion in the east-central subtropical oceans.

(c) Vertical lapse-rate advection

This term is the product of a vertical velocity and a vertical variation in a quantity, representing the vertical advection of the given quantity. Specifically, this term represents vertical advection *by the large-scale-averaged vertical velocity* of the *large-scale-averaged* lapse rate.

Like with horizontal advection, this term cannot create a vertical lapse-rate gradient if one does not already exist – it can only move (advect) an existing gradient. Because the trade-wind inversion is typically found in conjunction with the Hadley cell's descending branch, this term can help to push an existing inversion to lower altitudes. This influence is strongest where the large-scale descent is strongest and becomes weaker toward the Equator.

(d) Divergence term

This term can be perceived in two ways. In the form presented in (10), it is the product of the large-scale-averaged lapse rate and vertical variation in the large-scale-averaged vertical velocity. From continuity, however, the vertical variation in the large-scale-averaged vertical velocity is equivalent to the large-scale-averaged horizontal divergence field (where $\nabla \bullet \vec{v} = -\frac{\partial \overline{\omega}}{\partial p}$; such that divergence is associated with positive values of each quantity).

In the first context, the trade-wind inversion exists within a large-scale environment characterized by descent at the inversion's top and near-zero vertical velocity at its bottom. This pushes the top of the inversion downward without affecting the bottom of the inversion, consequently resulting in increased stability through the trade-wind inversion. In the second context, the trade-wind inversion exists within a large-scale environment characterized by divergence. Given that $\overline{\Gamma}$ is negative (since potential temperature increases as pressure decreases as you move upward through the inversion), divergence contributes to a stronger trade-wind inversion. Consequently, the two perspectives are equivalent in their assessments.

Since large-scale divergence is typically large in the Hadley cell's descending branch, this term is the primary contributor to trade-wind inversion formation and maintaining a strong inversion. This term's influence is largest in the eastern subtropical oceans and becomes weaker toward the west and toward the Equator.

(e) Turbulent vertical heat flux

This term reflects the large-scale-averaged effects of turbulent vertical motions in clouds. Potential temperature locally increases with height through an inversion. Conversely, clouds that form below the inversion are associated with both local ascending and descending motions. Together, however, a unified picture emerges: cloud-scale ascent mixes lower potential temperature upward and cloud-scale descent mixes higher potential temperature air downward. Physically, this weakens the trade-wind inversion.

Mathematically, we can use this information to identify the sign of $\overline{\omega'\theta'}$ through the inversion. As ascent is defined where $\omega' < 0$ and occurs where $\theta' < 0$, their product and subsequent large-scale-average are positive. Likewise, as descent is defined where $\omega' > 0$ and occurs where $\theta' > 0$, their product and subsequent large-scale-average are also positive. Thus, $\overline{\omega'\theta'} > 0$ through the inversion. Clouds are less prevalent both above and below the inversion, such that $\overline{\omega'\theta'} \approx 0$ at these altitudes.

The second partial derivative can be approximated using a finite-difference approximation as:

$$\frac{\partial^2}{\partial p^2}(\quad) = \frac{(\quad)_{top} + (\quad)_{bottom} - 2(\quad)_{mid}}{(\Delta p)^2} \tag{11}$$

Since we are evaluating (11) through the inversion where ( ) is largest, the $(\ )_{mid}$ term dominates the computation. Because this term is positive, $-2(\ )_{mid}$ is negative. Finally, because the denominator of (11) is positive-definite, the second derivative is negative. This implies $\frac{\partial \overline{\Gamma}}{\partial t} < 0$, such that turbulent vertical motions in clouds immediately below the inversion weaken the inversion over time.

(f) Heating terms

Ignoring the coefficients on the summation in this term, this term can be written as:

$$-\frac{\partial}{\partial p}\left(\Sigma \overline{H_i}\right) \tag{12}$$

where H_i is approximated by:

$$H_i = H_{con} + H_{ncon} + H_{evp} + H_{sen} + H_{rad} \tag{13}$$

The five terms of (13) represent latent heat release in thunderstorms, latent heat release in stratiform rains, latent heat release from evaporation, sensible heating, and radiative heating, respectively. If we assume that rainfall is infrequent in conjunction with the trade-wind inversion, we can neglect H_{con} and H_{ncon}, leaving us with only evaporation, sensible heating, and radiative processes.

Evaporation: By definition, evaporation is a cooling process ($H < 0$). It occurs beneath the inversion as drier environmental air is mixed into saturated clouds. This term is approximately zero at lower altitudes beneath the inversion. Consequently, H decreases upward below the inversion as pressure increases, such that its partial derivative with respect to p is positive and the entire term is negative given the leading negative sign on it in (12). Thus, evaporation weakens the trade-wind inversion. Physically, this results from cooling where it is relatively warm (through the inversion).

Sensible Heating: To gauge the impact of sensible heating on inversion strength, we must consider the temperature of the underlying surface, given here by the sea-surface temperature. In the eastern portion of an ocean basin, sea-surface temperature is less than that of the atmosphere. Here, sensible heating is downward (negative) as the air loses heat to the ocean. Intuitively, this promotes a near-surface inversion thus further stabilizes the environment. However, in western subtropical oceans, the increased distance from the cold ocean currents that characterize the eastern subtropical oceans results in the sea-surface temperature being greater than the atmosphere. Thus, sensible heating is upward (positive) as the air gains heat from the ocean. Intuitively, this destabilizes the environment. Mathematically,

- *Eastern subtropical oceans*: H negative at the surface, near-zero above. Thus, its partial derivative with respect to p is negative, such that the entire term is positive.
- *Western subtropical oceans*: H positive at the surface, near-zero above. Thus, its partial derivative with respect to p is positive, such that the entire term is negative.

Radiation: Consider again a cloud trapped just beneath the inversion's base. At the top of the cloud, heat is lost to space via both emission and reflection, constituting radiative cooling ($H < 0$). At the bottom of the cloud, radiation emitted from the cloud is largely trapped between it and the surface, constituting radiative warming ($H > 0$). Physically, this cools where it is relatively warm and warms where it is relatively cool, weakening the inversion. Mathematically, as H decreases as p decreases moving upward through the inversion, the entire term is negative giving the leading negative sign in (12) – supporting the physical interpretation that this term weakens the inversion.

Altogether, divergence and sensible heating (in the eastern subtropical oceans) act to build or strengthen an inversion, whereas cloud-scale turbulent vertical motions, evaporation, radiative processes, and sensible heating (in the western subtropical oceans) erode or weaken an inversion. Advection can locally strengthen or weaken an inversion but cannot create an inversion. The observed weakening of the trade-wind inversion with westward extent is largely a factor of weakened large-scale divergence with westward extent.

References

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