Synoptic Meteorology II: Frontogenesis and Frontolysis

Readings: Section 6.2 of *Midlatitude Synoptic Meteorology*.

Introduction

Frontogenesis, or the formation of a front, can be thought of as the strengthening of the cross-front temperature gradient. Its counterpart is *frontolysis*, or the decay of a front, which can be thought of as the weakening of the cross-front temperature gradient.

Frontogenesis and frontolysis can be assessed by using a *frontogenetical function*. Recall that a front is generally defined as a region of locally strong horizontal (potential) temperature gradient. Consequently, the frontogenetical function is a mathematical construct that defines frontogenesis and frontolysis in terms of how the magnitude of the horizontal potential-temperature gradient evolves following the motion on an isobaric surface, i.e.,

$$F = \frac{D}{Dt}\left|\nabla_p \theta\right| \tag{1}$$

Positive values of *F* denote that the magnitude of the horizontal potential-temperature gradient is becoming greater following the motion; negative values of *F* denote that the magnitude of the horizontal potential-temperature gradient is becoming weaker following the motion. While positive values of *F* do not necessarily mean that a front will *form*, and negative values of *F* do not necessarily mean that an existing front will *dissipate*, the frontogenetical function nevertheless provides a useful construct by which frontal evolution can be assessed.

As implied by the total derivative in (1), the most appropriate analysis of the frontogenetical function is in a reference frame following the motion – in this case, following the motion of the front. Such a reference frame is known as a *quasi-Lagrangian* reference frame.

Two-Dimensional Frontogenesis

As a first approximation, let us consider a "straight" frontal zone. Let us define a local *x*-axis that is parallel to the isentropes comprising this frontal zone, with warm air located 90° to the right of the positive *x*-axis. The *y*-axis is thus perpendicular to the isentropes, with the positive *y*-axis pointing toward cold air. The *u*-wind component is that which is found along the *x*-axis that we define, while the *v*-wind component is that which is found along the *y*-axis that we define. *Note that both the coordinate axes and thus the u and v wind components defined here often do not correspond to the cardinal directions (i.e., positive x-axis east, positive y-axis north)!*

We also assume that there are no variations in the wind field *along* the front such that $\partial u/\partial x$ and $\partial v/\partial x$ are both zero. This is not always a reasonable assumption; however, it does help to simplify the problem substantially.

Under these assumptions, (1) can be expressed as:

$$F = \frac{D}{Dt}\left(-\frac{\partial\theta}{\partial y}\right)_p \tag{2}$$

where the subscript of p in (2) denotes that it is evaluated on an isobaric surface. To simplify our subsequent derivation, we will make this subscript implicit from here on out; i.e., the equation we derive will implicitly be evaluated on isobaric surfaces.

With potential temperature decreasing along the positive y-axis (across the isentropes toward cold air), positive values of (2) are associated with the potential-temperature gradient becoming <u>larger</u> *following the motion*. Conversely, negative values of (2) are associated with this potential-temperature gradient becoming <u>smaller</u> *following the motion*.

If we expand (2) using the definition of the total derivative in isobaric coordinates, we obtain:

$$F = \frac{\partial}{\partial t}\left(-\frac{\partial\theta}{\partial y}\right) + u\frac{\partial}{\partial x}\left(-\frac{\partial\theta}{\partial y}\right) + v\frac{\partial}{\partial y}\left(-\frac{\partial\theta}{\partial y}\right) + \omega\frac{\partial}{\partial p}\left(-\frac{\partial\theta}{\partial y}\right) \tag{3}$$

Using the chain rule for partial derivatives and commuting the order of partial derivatives, we can write the following:

$$\frac{\partial}{\partial y}\left(u\frac{\partial\theta}{\partial x}\right) = \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} + u\frac{\partial}{\partial x}\left(\frac{\partial\theta}{\partial y}\right), \qquad \text{such that } u\frac{\partial}{\partial x}\left(-\frac{\partial\theta}{\partial y}\right) = \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial}{\partial y}\left(u\frac{\partial\theta}{\partial x}\right)$$

$$\frac{\partial}{\partial y}\left(v\frac{\partial\theta}{\partial y}\right) = \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + v\frac{\partial}{\partial y}\left(\frac{\partial\theta}{\partial y}\right), \qquad \text{such that } v\frac{\partial}{\partial y}\left(-\frac{\partial\theta}{\partial y}\right) = \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} - \frac{\partial}{\partial y}\left(v\frac{\partial\theta}{\partial y}\right)$$

$$\frac{\partial}{\partial y}\left(\omega\frac{\partial\theta}{\partial p}\right) = \frac{\partial \omega}{\partial y}\frac{\partial\theta}{\partial p} + \omega\frac{\partial}{\partial p}\left(\frac{\partial\theta}{\partial y}\right), \qquad \text{such that } \omega\frac{\partial}{\partial p}\left(-\frac{\partial\theta}{\partial y}\right) = \frac{\partial \omega}{\partial y}\frac{\partial\theta}{\partial p} - \frac{\partial}{\partial y}\left(\omega\frac{\partial\theta}{\partial p}\right)$$

Substituting into (3) and grouping like terms, we obtain:

$$F = \frac{\partial}{\partial t}\left(-\frac{\partial\theta}{\partial y}\right) + \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + \frac{\partial \omega}{\partial y}\frac{\partial\theta}{\partial p} - \frac{\partial}{\partial y}\left(u\frac{\partial\theta}{\partial x}\right) - \frac{\partial}{\partial y}\left(v\frac{\partial\theta}{\partial y}\right) - \frac{\partial}{\partial y}\left(\omega\frac{\partial\theta}{\partial p}\right) \tag{4}$$

If we commute the order of the partial derivatives in the first right-hand side term of (4) and place it with other terms like it, we obtain:

$$F = \frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y} + \frac{\partial \omega}{\partial y}\frac{\partial\theta}{\partial p} - \frac{\partial}{\partial y}\left(\frac{\partial\theta}{\partial t}\right) - \frac{\partial}{\partial y}\left(u\frac{\partial\theta}{\partial x}\right) - \frac{\partial}{\partial y}\left(v\frac{\partial\theta}{\partial y}\right) - \frac{\partial}{\partial y}\left(\omega\frac{\partial\theta}{\partial p}\right) \tag{5a}$$

Or, equivalently:

$$F = \frac{\partial u}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y} + \frac{\partial \omega}{\partial y}\frac{\partial \theta}{\partial p} - \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} + \omega\frac{\partial \theta}{\partial p}\right) \quad (5b)$$

The last term on the right-hand side of (5b) can be rewritten in terms of the total derivative, such that we finally obtain:

With some manipulation and commuting the order of the partial derivatives, we obtain:

$$F = \frac{\partial u}{\partial y}\frac{\partial \theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y} + \frac{\partial \omega}{\partial y}\frac{\partial \theta}{\partial p} - \frac{\partial}{\partial y}\left(\frac{D\theta}{Dt}\right) \quad (6)$$

There are four forcing terms on the right-hand side of (6). From left to right, these are known as the shearing, diffluence, tilting, and diabatic forcing terms. *Remember*: *u* and *v* are defined with respect to the local *x*- and *y*- axes and do <u>*not*</u> represent the zonal and meridional wind components! Let us now consider each term on the right-hand side of (6) in isolation.

Shearing Term

Expressing (6) in the context of the shearing term alone, we have:

$$F = \frac{\partial u}{\partial y}\frac{\partial \theta}{\partial x} \quad (7)$$

The shearing term is a function of two terms: (a) the change in the along-front wind between the cold and warm sides of the front (giving rise to the name “shearing” for this term) and (b) the change in potential temperature along the front.

For a truly “straight” frontal zone, in which the isentropes are parallel to the *x*-axis everywhere (and not just locally), this term is zero because there is no change in potential temperature along the front. A truly “straight” frontal zone does not always exist within the real atmosphere, however, such that this term is not always zero. The shearing term is largest when the magnitudes of its component terms are large: a large change in the along-front wind component across the front and/or a large change in potential temperature along the front.

Mathematically, the shearing term is frontogenetic when its components are of *like* sign:

- Potential temperature increasing along the positive *x*-axis ($\partial\theta/\partial x > 0$) as the *u* wind component becomes more positive along the positive *y*-axis ($\partial u/\partial y > 0$).
- Potential temperature decreasing along the positive *x*-axis ($\partial\theta/\partial x < 0$) as the *u* wind component becomes less positive along the positive *y*-axis ($\partial u/\partial y < 0$).

The latter of these two scenarios is depicted by Fig. 6.3 of the Lackmann text.

Likewise, the shearing term is frontolytic when its components are of *opposite* sign:

- Potential temperature increasing along the positive x-axis ($\partial\theta/\partial x > 0$) as the u wind component becomes less positive along the positive y-axis ($\partial u/\partial y < 0$).
- Potential temperature decreasing along the positive x-axis ($\partial\theta/\partial x < 0$) as the u wind component becomes more positive along the positive y-axis ($\partial u/\partial y > 0$).

The former of these two scenarios is depicted by Fig. 6.4 of the Lackmann text.

Diffluence Term

Expressing (6) in the context of the diffluence term alone, we have:

$$F = \frac{\partial v}{\partial y}\frac{\partial \theta}{\partial y} \tag{8}$$

In this simplified coordinate frame, diffluence is defined where $\partial v/\partial y > 0$, while confluence is defined where $\partial v/\partial y < 0$.

Because of how our coordinate axes are defined, with the positive y-axis pointing toward cold air, $\partial\theta/\partial y$ is negative. Thus, it is the sign of $\partial v/\partial y$ that determines the sign of F with respect to the diffluence term. We thus find that confluence ($\partial v/\partial y < 0$) is frontogenetic. This is manifest by cold potential temperature advection on the cold side of the frontal zone and warm potential temperature advection on the warm side of the frontal zone. Conversely, diffluence ($\partial v/\partial y < 0$) is frontolytic. This is manifest by warm potential temperature advection on the cold side of the frontal zone and cold potential temperature advection on the cold side of the frontal zone.

The diffluence term is largest when the magnitudes of the diffluence and/or potential-temperature gradient terms are large: a large change in the across-front wind component across the front and/or a large change in potential temperature across the front.

Confluence that contributes to frontogenesis is depicted in Fig. 6.5 of the Lackmann text. Note that this flow is convergent (along the positive y-axis) and associated with stretching deformation.

Tilting Term

Expressing (6) in the context of the tilting term alone, we have:

$$F = \frac{\partial \omega}{\partial y}\frac{\partial \theta}{\partial p} \tag{9}$$

This term is known as the tilting term because it represents the tilting of the vertical potential-temperature gradient $\partial\theta/\partial p$ into the horizontal direction by vertical motions (or, more specifically, horizontal gradients of vertical motions). Nominally, $\partial\theta/\partial p < 0$ as potential temperature increases ($\partial\theta > 0$) as you go up in the atmosphere ($\partial p < 0$) under statically stable conditions.

From (9), $\partial\omega/\partial y < 0$ strengthens the meridional potential-temperature gradient. Physically, this is manifest by ascent ($\omega < 0$) on the cold side of the frontal zone and descent ($\omega > 0$) on the warm side of the frontal zone. Stated differently, the adiabatic cooling (expansion) associated with ascent and adiabatic warming (compression) associated with descent intensifies the meridional potential-temperature gradient by cooling where it is cool and warming where it is warm. This scenario is depicted by Fig. 6.6 of the Lackmann text.

Conversely, $\partial\omega/\partial y > 0$ weakens the meridional potential-temperature gradient. Physically, this is manifest by ascent on the warm side of the frontal zone and descent on the cold side of the frontal zone. Stated differently, the adiabatic cooling (expansion) associated with ascent and adiabatic warming (compression) associated with descent weakens the meridional potential-temperature gradient by warming where it is cool and cooling where it is warm.

In the lower troposphere, where vertical motions are relatively weak, the tilting term is relatively small except in regions of sloped terrain. The tilting term can be quite important in the middle to upper troposphere, however, where vertical motions are stronger.

The diffluence term and tilting term can be at odds with one another. An example of this is given by the entrance region of an upper tropospheric jet streak. The flow is often confluent within the entrance region of a jet streak. As a result, the diffluence term implies frontogenesis. However, the entrance region of a jet streak is also characterized by ascent in the right entrance region and descent in the left entrance region. Because of the definition of the thermal wind, the air on the right side of the jet must be warmer than that on the left side of the jet. Consequently, ascent occurs where it is warm and descent occurs where it is cold, indicative of what is known as a *thermally direct* circulation. As a result, the tilting term implies frontolysis. Similarly, in the exit region of an upper tropospheric jet streak, the diffluence term implies frontolysis whereas the tilting term implies frontogenesis. The proof of this statement is left to the interested reader.

As with the previous terms, the tilting term is largest when the magnitudes of its components are largest: a large gradient in vertical velocity across the front and/or strong static stability (potential temperature rapidly increasing with height).

Diabatic Term

Expressing (6) in the context of the diabatic term alone, we have:

$$F = \frac{\partial}{\partial y}\left(\frac{D\theta}{Dt}\right) \tag{10}$$

This term represents meridional variability in diabatic heating, as changes in potential temperature following the motion occur only when there are active diabatic processes. If you warm where it is already warm and cool where it is already cool, the meridional potential-temperature gradient will become stronger. This is associated with $D\theta/Dt > 0$ in the warm air to the south and $D\theta/Dt < 0$ in the cold air to the north, such that $\partial(D\theta/Dt)/\partial y < 0$ and, subsequently, $F > 0$. The diabatic term is largest when the gradient of diabatic heating across the front is largest.

Physically, this can occur for the situation where there is strong sensible heating of the lower troposphere by insolation on the warm side of the front and no sensible heating and modest evaporational cooling of the lower troposphere beneath a cloud deck on the cold side of the front. This is depicted by Fig. 6.7 of the Lackmann text. At night, however, the same situation can be frontolytic: strong sensible heating on the warm side of the front is replaced by radiative cooling that is absent on the cold side of the front due to the insulating effects of cloud cover.

Another situation in which the diabatic term can be frontogenetic is that associated with across-front variability in snow cover. Snow-covered ground on the cold side of the front will reinforce and/or intensify the front due to the strong reflective and radiational characteristics of snow. These are but a few physical examples of the diabatic term's influences upon frontogenesis. In the lower troposphere, gradients in sensible heating contribute most strongly to the diabatic term. Aloft, gradients in latent heating contribute most strongly to the diabatic term.

Further Examination of the Shearing and Diffluence Terms

Let us no longer assume that the wind field does not change along the front itself. Therefore, (1) becomes:

$$F = \frac{D}{Dt}\left|\nabla_p \theta\right| = \frac{D}{Dt}\left(\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right)^{\frac{1}{2}}\right) \tag{11}$$

Note that we wrote the final expression in (11) in terms of a quantity raised to the ½ power rather than in terms of the square root. This is to allow us to use the following relationship:

$$\frac{D}{Dt}(f^n) = nf^{n-1}\frac{D}{Dt}(f) \tag{12}$$

If we let $f = \left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right)$ and $n = ½$, we obtain the following:

$$\frac{D}{Dt}\left|\nabla_p\theta\right| = \frac{1}{2}\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right)^{-\frac{1}{2}}\frac{D}{Dt}\left(\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right)\right) \tag{13a}$$

Or, equivalently:

$$\frac{D}{Dt}\left|\nabla_p\theta\right| = \frac{1}{2\left|\nabla_p\theta\right|}\frac{D}{Dt}\left(\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right)\right) \tag{13b}$$

Note that we have dropped the *p* subscript everywhere except on the gradient operator. It remains implicit throughout (13a) and (13b) as well as subsequent equations, however.

Next, we can expand the right-hand side of (13b) using the chain rule, where:

$$\frac{D}{Dt}\left(\left(\frac{\partial\theta}{\partial x}\right)^2\right) = \frac{\partial\theta}{\partial x}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial x}\right) + \frac{\partial\theta}{\partial x}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial x}\right) = 2\frac{\partial\theta}{\partial x}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial x}\right)$$

$$\frac{D}{Dt}\left(\left(\frac{\partial\theta}{\partial y}\right)^2\right) = \frac{\partial\theta}{\partial y}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial y}\right) + \frac{\partial\theta}{\partial y}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial y}\right) = 2\frac{\partial\theta}{\partial y}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial y}\right)$$

Substituting these into (13b), we obtain:

$$F = \frac{D}{Dt}\left|\nabla_p\theta\right| = \frac{1}{\left|\nabla_p\theta\right|}\left[\frac{\partial\theta}{\partial x}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial x}\right) + \frac{\partial\theta}{\partial y}\frac{D}{Dt}\left(\frac{\partial\theta}{\partial y}\right)\right] \tag{14}$$

Next, we expand the two total derivative terms inside of the brackets in (14), neglecting the terms involving ω (no tilting) and diabatic heating. Manipulating and simplifying the result allows us to obtain:

$$F = \frac{1}{\left|\nabla_p\theta\right|}\left[-\left(\frac{\partial\theta}{\partial x}\right)^2\frac{\partial u}{\partial x} - \frac{\partial\theta}{\partial x}\frac{\partial\theta}{\partial y}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) - \left(\frac{\partial\theta}{\partial y}\right)^2\frac{\partial v}{\partial y}\right] \tag{15}$$

Horizontal divergence, stretching deformation, and shearing deformation are given by:

$$\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{16a}$$

$$D_{st} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \tag{16b}$$

$$D_{sh} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \tag{16c}$$

The total deformation is equal to the magnitude of the stretching and shearing deformation, i.e.,

$$D = \sqrt{D_{st}^2 + D_{sh}^2} \tag{17}$$

An inspection of (15) shows that the second forcing term can be written directly in terms of the shearing deformation given by (16c). If we solve the system of equations given by (16a) and (16b) for each of *∂u/∂x* and *∂v/∂y*, the other two terms of (15) can be written in terms of horizontal divergence and stretching deformation. Doing so, we obtain:

$$F = \frac{1}{\left|\nabla_p \theta\right|}\left[-\left(\frac{\partial \theta}{\partial x}\right)^2\left(\frac{1}{2}(\delta + D_{st})\right) - \frac{\partial \theta}{\partial x}\frac{\partial \theta}{\partial y}D_{sh} - \left(\frac{\partial \theta}{\partial y}\right)^2\left(\frac{1}{2}(\delta - D_{st})\right)\right] \tag{18}$$

The advantage of (18) over (6) is that it explicitly expresses the effects of confluence and diffluence in terms of the contributions to each from both horizontal divergence and deformation.

The coordinate system can be rotated such that the *x*-axis lies along either the *axis of dilatation* or the *axis of contraction*. The axis of dilatation and axis of contraction are depicted graphically in Fig. 1 below for a pure deformation flow. If the *x*-axis is rotated to lie along the axis of dilatation, the *y*-axis lies along the axis of contraction, and vice versa.

In this coordinate system, (18) can be re-written as:

$$F = \frac{1}{2}\left|\nabla_p \theta\right|[D\, cos(2b) - \delta] \tag{19}$$

In (19), *b* is the angle between the isentropes and the axis of dilatation. *D* is given by (17), except in the new coordinate system defined relative to the axes of dilatation and contraction.

In (19), let us first consider the effects of deformation upon frontogenesis, i.e.,

$$F \approx \frac{1}{2}\left|\nabla_p \theta\right|[D\, cos(2b)] \tag{20}$$

When the axis of dilatation lies within a 45° angle of the isentropes, deformation is a frontogenetical process. This corresponds to the left-most panel of Fig. 1 below and to Fig. 6.9c,d of the Lackmann text (noting that *b* = 0° in this example).

Conversely, when the axis of dilatation lies between a 45° and 90° angle of the isentropes, deformation is a frontolytic process. This corresponds to the right-most panel of Fig. 1 above and to Fig. 6.9a,b of the Lackmann text (noting that *b* = 90° in this example).

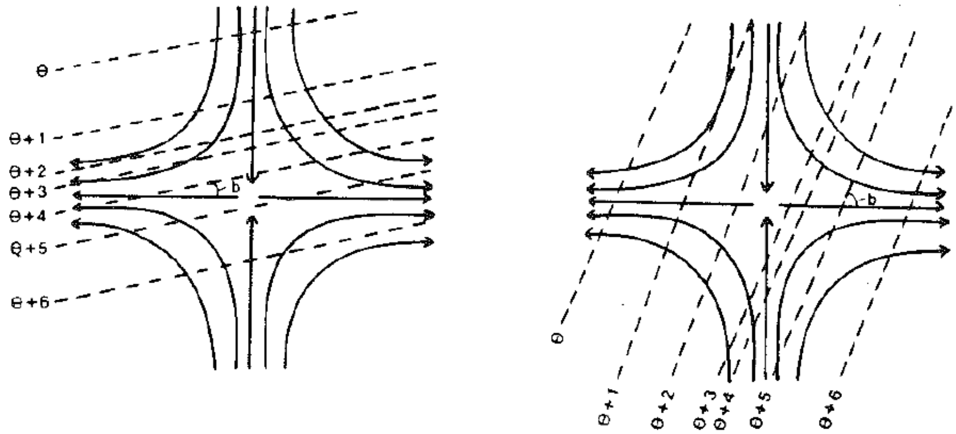

Figure 1. Pure deformation flow (black streamlines) with the *x*-axis aligned along the axis of dilatation and the *y*-axis aligned along the axis of contraction. Isentropes are depicted by the black dashed lines. In the left (right) panel, the angle between the isentropes and the axis of dilatation is less (greater) than 45°, a frontogenetic (frontolytic) situation. This can be confirmed by visually interpolating how the flow will cause the horizontal potential-temperature gradient to evolve with time. Figure reproduced from *Mid-Latitude Synoptic-Dynamic Meteorology* (Vol. II) by H. Bluestein, their Fig. 2.13.

Next, let us return to (19) to consider the effects of divergence upon frontogenesis, i.e.,

$$F \approx -\frac{1}{2}\left|\nabla_p \theta\right|\delta \tag{21}$$

Since $\delta > 0$ for divergence and $\delta < 0$ for convergence, convergent flow is frontogenetic whereas divergent flow is frontolytic, no matter the horizontal potential-temperature gradient's orientation.

In the above, we considered the effects of deformation and horizontal divergence on frontogenesis. It may be natural, therefore, to ask if vertical vorticity (or rotation) can result in frontogenesis or frontolysis. Rotation can only rotate the gradient. However, rotation can change the angle of the gradient with respect to the axes of dilatation and contraction, indirectly allowing for frontogenesis or frontolysis. Deformation also can change the angle of the gradient with respect to the axes of dilatation and contraction, although this is perhaps not as easy to visualize.

Likewise, note that we have, to this point, considered each component of the flow – deformation, horizontal divergence, and vertical vorticity – in isolation. In the real atmosphere, the flow contains all three components; it is not exclusively comprised of only rotation, divergence, or deformation. Thus, these processes act in concert with one another, sometimes in a constructive way, sometimes in a conflicting way, to impact frontogenesis and frontolysis.