Synoptic Meteorology II: Application of the Quasi-Geostrophic Vorticity Equation

Advection Processes

To begin our interpretation of (20), we wish to consider only the first two terms on its right-hand side, those related to horizontal advection of geostrophic relative vorticity and planetary vorticity. To do so, we will utilize a schematic of an idealized synoptic-scale trough/ridge pattern, as given by Fig. 1 below.

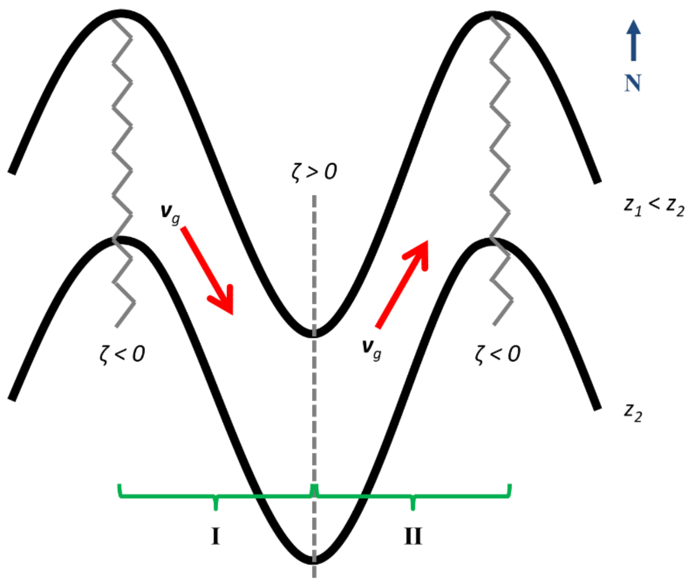

Figure 1. Schematic 500 hPa geopotential height field used to evaluate changes in the geostrophic relative vorticity resulting from advection of both geostrophic relative and planetary vorticity. See the text below for further details.

To the east of the base of the trough (region II in Fig. 1), the meridional flow is from south to north ($v_g > 0$). Because f increases with increasing latitude in the Northern Hemisphere, and because β is defined as the derivative of f with respect to latitude (i.e., $\beta = \partial f/\partial y$), β is positive. With the leading negative sign, the planetary advection term is negative, implying a decrease in geostrophic relative vorticity. However, the large-scale flow is directed from the base of the trough, where the geostrophic relative vorticity is cyclonic (positive in the Northern Hemisphere). This implies an increase in geostrophic relative vorticity, and thus the two advection terms are of opposing sign.

Conversely, to the west of the base of the trough (region I in Figure 1), the meridional flow is from north to south ($v_g < 0$). β remains positive, such that with the leading negative sign, the planetary advection term is positive, implying an increase in geostrophic relative vorticity. Likewise, the large-scale flow is directed from the apex of the ridge, where the geostrophic relative vorticity is anticyclonic (negative in the Northern Hemisphere). This implies a decrease in geostrophic relative vorticity, and thus the two advection terms are again of opposing sign.

The two advection terms can be used to describe only the ***motion*** – and **not** the *amplification or deamplification* – of the synoptic-scale, midlatitude pattern. Why? First, recall that the geostrophic wind blows parallel to the isohypses. For a trough or ridge to amplify or deamplify, the geostrophic wind would need to have a non-zero component blowing perpendicular to the contours of constant geopotential height. Since it does not, it cannot change the amplitude of the pattern. Second, recall that the geopotential height field and geostrophic relative vorticity are inextricably linked through (14). If geostrophic relative vorticity is advected from one location to another by the geostrophic wind, the geopotential height and the geostrophic wind will change to match.

To better illustrate this, let us examine the advection terms in the base of the trough (at the junction of I and II) in Fig. 1. The β term in (20) is zero because v_g is zero. The geostrophic wind is directed from west to east, locally parallel to the geopotential height contours. Thus, the geostrophic relative vorticity advection term advects (or transports) cyclonic geostrophic relative vorticity eastward. Because of (14), however, the geopotential height field also shifts eastward so that the geopotential height minimum remains tied to the geostrophic relative vorticity maximum. The geostrophic wind remains easterly, and the process continues. The same arguments can be made in the apex of one of the ridges in Fig. 1.

How do we know which of the two advection terms is of larger magnitude? While the derivation necessary to understand this is beyond the scope of this class, the results of doing so indicate:

- For *shortwave* troughs, or those of zonal (east-west) extent less than approximately 3,000 km, the geostrophic horizontal advection of geostrophic relative vorticity dominates over the geostrophic meridional advection of planetary vorticity. Thus, these features generally move eastward with the westerly large-scale flow.

- For *longwave* troughs, or those of zonal extent greater than approximately 10,000 km, the geostrophic meridional advection of planetary vorticity dominates over the geostrophic horizontal advection of geostrophic relative vorticity. Because this results in anticyclonic geostrophic relative vorticity advection east of and cyclonic geostrophic relative vorticity advection west of a trough, longwave troughs move westward, or *retrogress*, against the westerly large-scale flow.

- Troughs of intermediate zonal extent (between 3,000-10,000 km) tend to move eastward, albeit at a rate of speed slower to much slower than that of the westerly large-scale flow.

In observations, however, longwave troughs tend to remain relatively stationary rather than retrogress. There are many reasons why this may be the case, many of which are not explicitly considered in the context of quasi-geostrophic theory: topographic influences, large-scale thermal contrasts (e.g., land versus water), and non-linear interaction(s) with shortwave troughs rotating around the base of the longwave trough. Thus, it is helpful to remember that quasi-geostrophic theory is merely an *approximation* to the real atmosphere!

Inclusion of Vertical Motion

In the above, we neglected the third term on the right-hand side of (20), or that associated with the vertical stretching of planetary vorticity. This process can be viewed in the context of a figure skater performing a spin on ice. As the skater begins their spin, they are low to the ground. Their rate of rotation is relatively slow (or small). However, as they continue their spin, they bring (or converge) their arms inward, becoming more upright – or stretched vertically – as they do so. This intensifies their rate of rotation.

Applying this concept to (20), consider the case where there is *rising* motion throughout the troposphere that is maximized within the middle troposphere (such as near 500 hPa). Because $\omega < 0$ denotes rising motion, $\partial\omega$ is negative in the lower to middle troposphere. Likewise, ∂p is negative because pressure decreases with increasing altitude. Thus, the last term on the right-hand side of (20) is positive, implying an increase in the local magnitude of the geostrophic relative vorticity.

The analogy and application work in the inverse, as well. Consider the case where there is *descending* motion throughout the troposphere that is maximized within the middle troposphere. Because $\omega > 0$ denotes descending motion, $\partial\omega$ is positive in the lower to middle troposphere. As before, ∂p is negative because pressure decreases with increasing altitude. Thus, the last term on the right-hand side of (20) is negative. This implies a decrease in the local magnitude of the geostrophic relative vorticity as the vortex tube (or ice skater) is compressed toward the ground.

Thus, when considering the local evolution of geostrophic relative vorticity, both horizontal advection as well as vertical motions must be considered! From the continuity equation, these vertical motions are exclusively a function of the ageostrophic wind, which in and of itself is tied to parcel accelerations. As we study the evolution of synoptic-scale weather systems, we will see precisely how horizontal advection and vertical motions work in concert with one another to influence the evolutions of these phenomena.