

## Synoptic Meteorology II: Synoptic Development – The Pettersen-Sutcliffe Framework

**Readings:** Sections 5.3.3 through 5.3.5 of *Midlatitude Synoptic Meteorology*.

### Introduction

To this point in the semester, we have concentrated on understanding and applying four equations: the quasi-geostrophic vorticity, height-tendency, and omega equations, as well as the Q-vector form of the omega equation. However, each equation is typically applied at mid-levels. While the movement and evolution of troughs and ridges above the surface is important, we are particularly interested the movement and evolution of surface features. In other words, how do surface cyclones and anticyclones evolve in response to synoptic-scale quasi-geostrophic forcing?

The quasi-geostrophic vorticity and omega equations form a system that can be used to interpret and describe the behavior of midlatitude synoptic-scale weather systems. This is known as *Pettersen-Sutcliffe development theory*. Here, we aim to describe how these equations may be used to assess the synoptic-scale conditions that promote cyclone development (or cyclogenesis).

### Obtaining the Pettersen-Sutcliffe Development Equation

Since geostrophic relative vorticity  $\zeta_g$  (by definition) and potential temperature  $\theta$  (by application of the hydrostatic equation and Poisson's relation) can be written in terms of the geopotential height  $\Phi$ , the quasi-geostrophic vorticity and omega equations form a system of two equations for two unknowns ( $\Phi$  and  $\omega$ ), presuming that we know or can estimate the diabatic heating rate  $dQ/dt$ .

We start our derivation by recalling the quasi-geostrophic vorticity equation, with the local change and advection terms combined into the total derivative:

$$\frac{D_g}{Dt}(\zeta_g) = -\beta v_g + f_0 \frac{\partial \omega}{\partial p} - K \zeta_g \quad (1)$$

The effect of the  $-\beta v_g$  term, while non-zero, is quite small on the synoptic-scale. For typical values of  $\beta$  and  $v_g$ , we find this term to contribute to an approximate  $1 \times 10^{-5} \text{ s}^{-1}$  increase in geostrophic relative vorticity per day. Observed synoptic-scale midlatitude cyclones tend to deepen at a rate approximately one order of magnitude larger than this value. Thus, we neglect this term. Friction, as expressed by the third term on the right-hand side of (1), weakens both synoptic-scale cyclones and anticyclones. If we then rewrite (1) without these terms, we obtain:

$$\frac{D_g}{Dt}(\zeta_g) \approx f_0 \frac{\partial \omega}{\partial p}$$

Thus, for *cyclone development*, the stretching term in the quasi-geostrophic vorticity equation must be positive at and near the surface. Since  $f_0$  is positive in the Northern Hemisphere, we require that  $\partial\omega/\partial p$  be positive near the surface.

When does this occur? First, we must assume that the vertical motion vanishes at the surface (i.e.,  $\omega_{sfc} = 0$ ). This implies that there is no vertical motion across the rigid surface of the ground. Since  $\partial p < 0$ ,  $\partial\omega$  must also be negative. With  $\omega_{sfc} = 0$ ,  $\omega_{aloft} < 0$ . This means that there must be middle tropospheric synoptic-scale ascent for there to be surface cyclone development!

We use the omega equation to evaluate the conditions resulting in middle tropospheric synoptic-scale ascent. This requires manipulating the omega equation, given by (2) below, to determine the value of  $\partial\omega/\partial p$  at the surface ( $p = p_{sfc}$ ).

$$\begin{aligned} \sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} \\ = -f_0 \frac{\partial}{\partial p} \left( -\vec{v}_g \cdot \nabla (\zeta_g + f) \right) - h \nabla^2 (-\vec{v}_g \cdot \nabla \theta) + f_0 \frac{\partial}{\partial p} (K \zeta_g) \\ - \frac{R}{pc_p} \nabla^2 \left( \frac{dQ}{dt} \right) \end{aligned} \quad (2)$$

In the quasi-geostrophic omega equation, we have a term that looks similar to  $\partial\omega/\partial p$ , as given by  $\partial^2\omega/\partial p^2$ . We wish to rewrite (2) with only this term on the left-hand side of the equation. We also wish to express the geostrophic potential-temperature advection term in terms of the geopotential height  $\Phi$  via use of the form of the hydrostatic relationship given by:

$$\frac{\partial \Phi}{\partial p} = -h\theta \quad (3)$$

Doing so, while also neglecting the friction term, we obtain:

$$f_0^2 \frac{\partial^2 \omega}{\partial p^2} = -f_0 \frac{\partial}{\partial p} \left( -\vec{v}_g \cdot \nabla (\zeta_g + f) \right) - \nabla^2 \left( -\vec{v}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \right) - \sigma \nabla^2 \omega - \frac{R}{pc_p} \nabla^2 \left( \frac{dQ}{dt} \right) \quad (4)$$

To get an expression for  $\partial\omega/\partial p$ , we integrate (4) with respect to  $p$ . We do so from the surface ( $p = p_{sfc}$ ) to the level of non-divergence (LND;  $p = p_{LND}$ ), which is typically in the middle troposphere (as we learned last semester). Performing this integration, we obtain:

$$\begin{aligned} f_0^2 \frac{\partial \omega}{\partial p} \Big|_{p_{LND}} - f_0^2 \frac{\partial \omega}{\partial p} \Big|_{p_{sfc}} = -f_0 \left( -\vec{v}_g \cdot \nabla (\zeta_g + f) \right) \Big|_{p_{LND}} + f_0 \left( -\vec{v}_g \cdot \nabla (\zeta_g + f) \right) \Big|_{p_{sfc}} - \\ \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left( -\vec{v}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \right) dp - \int_{p_{sfc}}^{p_{LND}} \left[ \sigma \nabla^2 \omega + \frac{R}{pc_p} \nabla^2 \left( \frac{dQ}{dt} \right) \right] dp \end{aligned} \quad (5)$$

The first term on the left-hand side of (5) is zero: the  $\partial\omega/\partial p$  term equals the divergence, which is zero at the LND. Similarly, we neglect the second term on the right side of (5) by presuming that geostrophic absolute-vorticity advection at the surface is small. With this in mind, (5) becomes:

$$-f_0^2 \frac{\partial\omega}{\partial p} \Big|_{p_{sfc}} = -f_0 \left( -\vec{v}_g \cdot \nabla(\zeta_g + f) \right) \Big|_{p_{LND}} - \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left( -\vec{v}_g \cdot \nabla \left( -\frac{\partial\Phi}{\partial p} \right) \right) dp - \int_{p_{sfc}}^{p_{LND}} \left[ \sigma \nabla^2 \omega + \frac{R}{pc_p} \nabla^2 \left( \frac{dQ}{dt} \right) \right] dp \quad (6)$$

If we substitute the simplified form of the quasi-geostrophic vorticity equation (1) into (6) while multiplying all terms by -1, we obtain:

$$f_0 \frac{D_g}{Dt} (\zeta_g) \Big|_{p_{sfc}} = f_0 \left( -\vec{v}_g \cdot \nabla(\zeta_g + f) \right) \Big|_{p_{LND}} + \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left( -\vec{v}_g \cdot \nabla \left( -\frac{\partial\Phi}{\partial p} \right) \right) dp + \int_{p_{sfc}}^{p_{LND}} \left[ \sigma \nabla^2 \omega + \frac{R}{pc_p} \nabla^2 \left( \frac{dQ}{dt} \right) \right] dp \quad (7)$$

(7) is what we call the *Pettersen-Sutcliffe Development Equation* and relates the change in surface geostrophic relative vorticity following the geostrophic flow to three forcing terms. We now wish to consider the contributions from each of these terms to surface cyclone development in isolation.

### Interpretation of the Pettersen-Sutcliffe Development Equation

Let us first consider the case where the only forcing from (7) is the geostrophic absolute-vorticity advection term, i.e.,

$$f_0 \frac{D_g}{Dt} (\zeta_g) \Big|_{p_{sfc}} \approx f_0 \left( -\vec{v}_g \cdot \nabla(\zeta_g + f) \right) \Big|_{p_{LND}} \quad (8)$$

For cyclone development to occur,  $\left( -\vec{v}_g \cdot \nabla(\zeta_g + f) \right) \Big|_{p_{LND}}$  must be positive. Thus, *there must be cyclonic geostrophic absolute-vorticity advection occurring at the LND for cyclone development!*

Let us now consider the case where the only forcing from (7) is the term containing static stability and diabatic heating, i.e.,

$$\begin{aligned}
f_0 \frac{D_g}{Dt} (\zeta_g) \Big|_{p_{sfc}} &\approx \int_{p_{sfc}}^{p_{LND}} \left[ \sigma \nabla^2 \omega + \frac{R}{p c_p} \nabla^2 \left( \frac{dQ}{dt} \right) \right] dp \\
&= \int_{p_{sfc}}^{p_{LND}} \sigma \nabla^2 \omega dp + \int_{p_{sfc}}^{p_{LND}} \frac{R}{p c_p} \nabla^2 \left( \frac{dQ}{dt} \right) dp
\end{aligned} \tag{9}$$

Let us first consider the static stability term, that involving  $\sigma$ . Recall that  $\sigma$  is a measure of the dry static stability, where  $\sigma < 0$  implies static instability (potential temperature decreasing with height) and  $\sigma > 0$  implies static stability (potential temperature increasing with height). We let  $\sigma \nabla^2 \omega$  be approximated by a layer-mean value  $\overline{\sigma \nabla^2 \omega}$  such that it is no longer a function of  $p$ . Thus,

$$\int_{p_{sfc}}^{p_{LND}} \sigma \nabla^2 \omega dp \approx \overline{\sigma \nabla^2 \omega} \int_{p_{sfc}}^{p_{LND}} dp = \overline{\sigma \nabla^2 \omega} (p_{LND} - p_{sfc}) \tag{10}$$

Since  $p_{LND} - p_{sfc}$  is negative, for surface cyclone development to occur,  $\overline{\sigma \nabla^2 \omega}$  must be negative. Recall from (1) that there must be layer-mean ascent ( $\overline{\omega} < 0$ ) for surface cyclone development to occur. Since  $\nabla^2 \omega \propto -\omega$ , the Laplacian term is positive. Thus, for  $\overline{\sigma \nabla^2 \omega}$  to be negative,  $\sigma$  must be negative, signifying static instability. However, dry static instability is not often present, such that  $\sigma$  is most often positive. Thus, *ascent results in a decrease in cyclonic geostrophic relative vorticity at the surface over time, a **cyclolytic** situation!*

Next, consider the diabatic heating term. Presume that the diabatic heating can also be represented by a layer mean value, such that it is no longer a function of  $p$ . Thus, we write:

$$f_0 \frac{D_g}{Dt} (\zeta_g) \Big|_{p_{sfc}} \approx \frac{R}{c_p} \nabla^2 \left( \overline{\frac{dQ}{dt}} \right) \int_{p_{sfc}}^{p_{LND}} d \ln(p) = \frac{R}{c_p} \nabla^2 \left( \overline{\frac{dQ}{dt}} \right) \ln \left( \frac{p_{LND}}{p_{sfc}} \right) \tag{11}$$

Since  $p_{LND} < p_{sfc}$ , the natural logarithm term is negative. Thus, for surface cyclone development, we require that  $\nabla^2 \left( \overline{\frac{dQ}{dt}} \right) < 0$ . Since  $\nabla^2 \left( \overline{\frac{dQ}{dt}} \right) \propto -\frac{\overline{dQ}}{dt}$ , this requires that  $\frac{\overline{dQ}}{dt} > 0$ . Thus, we find that *diabatic warming contributes positively to surface cyclone development!* This also makes physical sense: layer-mean diabatic warming increases the thickness of that layer, which is associated with a downward extension of the isobaric surfaces in the bottom half of this layer that corresponds to lower heights and pressure at such altitudes.

It is worth noting that diabatic warming generally acts to decrease the static stability  $\sigma$ , particularly in the layer above that in which the diabatic warming is maximized. As a result, diabatic warming has both direct and indirect positive contributions to surface cyclone development. This allows us to state that the presence of moisture, associated with diabatic heating such as manifest through latent heat release, can substantially aid surface cyclone development!

Finally, consider the case where the only forcing from (7) is the geostrophic advection of the partial derivative of geopotential height with respect to pressure, i.e.,

$$f_0 \frac{Dg}{Dt} (\zeta_g) \Big|_{p_{sfc}} \approx \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left( -\vec{v}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \right) dp \quad (12)$$

If we assume that the *direction* of the potential temperature gradient  $\nabla \theta$  is constant with height (recalling from the hydrostatic equation that  $\nabla(h\theta) = \nabla \left( -\frac{\partial \Phi}{\partial p} \right)$ ), such that isotherms are oriented in the same direction on each isobaric level from the surface to the LND, we obtain:

$$-\vec{v}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \approx -\vec{v}_g \Big|_{p_{sfc}} \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \quad (13)$$

Since  $\vec{v}_g \Big|_{p_{sfc}}$  is constant with respect to pressure, this allows us to write:

$$f_0 \frac{Dg}{Dt} (\zeta_g) \Big|_{p_{sfc}} \approx \nabla^2 \left( -\vec{v}_g \Big|_{p_{sfc}} \cdot \nabla \left( -\int_{p_{sfc}}^{p_{LND}} \left( \frac{\partial \Phi}{\partial p} dp \right) \right) \right) \quad (14)$$

If we take the integral represented in (14), we obtain:

$$f_0 \frac{Dg}{Dt} (\zeta_g) \Big|_{p_{sfc}} \approx -\nabla^2 \left( -\vec{v}_g \Big|_{p_{sfc}} \cdot \nabla \left( \Phi_{p_{LND}} - \Phi_{p_{sfc}} \right) \right) \quad (15)$$

In (15),  $\Phi_{p_{LND}} - \Phi_{p_{sfc}}$  is the thickness of the layer between  $p = p_{LND}$  and  $p = p_{sfc}$ . The hypsometric equation thus allows us to alternatively express (15) as:

$$f_0 \frac{Dg}{Dt} (\zeta_g) \Big|_{p_{sfc}} \approx -\nabla^2 \left( -\vec{v}_g \Big|_{p_{sfc}} \cdot \nabla \bar{\theta} \right) \quad (16)$$

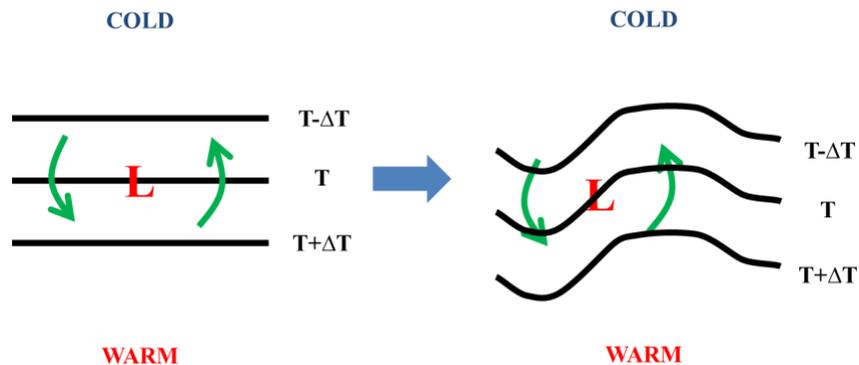
In (16), the overbar on  $\theta$  indicates a vertical average between  $p_{LND}$  and  $p_{sfc}$ .

For surface cyclone development, given the sign convention on the Laplacian operator in (16), we desire to find where  $-\vec{v}_g \Big|_{p_{sfc}} \cdot \nabla \bar{\theta} > 0$ . This term represents the advection of layer-mean potential temperature by the surface geostrophic wind. Based upon the sign convention on advection, we find that *this – and, thus, surface cyclone development – occurs when there is warm layer-mean potential temperature advection!*

Since this term is non-zero only when there is a horizontal gradient of potential temperature, which is a measure of *baroclinicity*, we expect that cyclone development occurs in the presence of non-zero baroclinicity (i.e., in the presence of a horizontal potential temperature gradient).

One caveat regarding the interpretation of this term, however. Because of the assumption that the direction of the horizontal potential temperature gradient is constant with height, we approximate the advection term with the advection by the surface geostrophic wind. However, the surface geostrophic wind vanishes at the center of a cyclone (or anticyclone, for that matter). Thus, there is no net advection over the center of a surface cyclone in as much as this assumption – as well as all those inherent to the quasi-geostrophic system – holds.

Thus, we state that this term is not responsible for surface cyclone *intensity changes* but, rather, is responsible primarily for surface cyclone *motion*. As a surface cyclone will generally track towards regions favoring development, surface cyclones move toward areas of layer-mean warm potential temperature advection and away from areas of layer-mean cold potential temperature advection. In the Northern Hemisphere, for cyclonic rotation, layer-mean warm potential temperature advection is typically found to its north and east whereas layer-mean cold potential temperature advection is typically found to its south and west. This is depicted in Fig. 1 below.



**Figure 1.** Idealized schematic of the rotation of layer-mean isotherms (black lines) by the cyclonic geostrophic flow associated with a surface cyclone (green lines). The cyclonic geostrophic flow results in warm (cold) advection to the north/east (south/west).

### The Concept of “Self-Development”

If we consider the interpretations of each forcing term together, we can state:

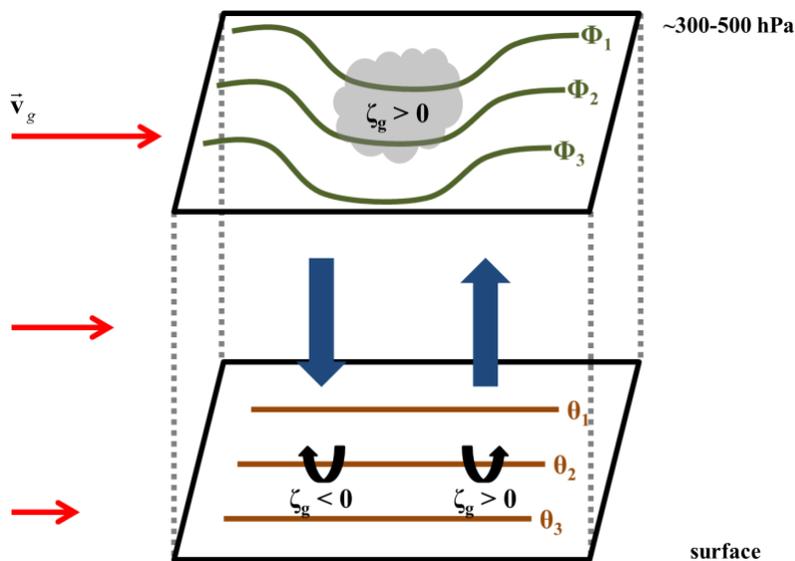
- Surface cyclone development occurs if sufficiently large cyclonic geostrophic absolute-vorticity advection in the mid-upper troposphere occurs atop a surface baroclinic zone.

- When there is diabatic warming, such as is often found with latent heat release associated with condensation and precipitation, such development can be more rapid and/or intense.
- Surface cyclones move toward regions of development, largely manifest through thermal advection patterns associated with the cyclone itself.

These findings provide the basic framework for what is known as the “self-development” paradigm for cyclone development. The remainder of this lecture will illustrate this in the context of the full life cycle of a synoptic-scale midlatitude surface cyclone. Note that in the following, we assume that there is westerly vertical wind shear in approximate thermal wind balance with the horizontal temperature gradient and that we are in a frame of reference moving with the system.

*Step 1: Surface Cyclogenesis*

In Step 1, an upper-tropospheric trough approaches a lower-tropospheric baroclinic zone, such as that associated with a remnant frontal boundary. There is cyclonic geostrophic absolute-vorticity advection ahead of the trough and anticyclonic geostrophic absolute-vorticity advection behind the trough. If we presume that the trough is at or near the LND, then from our interpretation of (7), there should be ascent and surface cyclone development ahead of the trough and descent and surface anticyclone development behind the trough. This leads to initial cyclone and anticyclone development, as conceptualized in Fig. 2 below.



**Figure 2.** Idealized schematic of the upper- and lower-tropospheric pattern and accompanying forcings associated with the genesis stage of a synoptic-scale surface cyclone. The upper tropospheric pattern is at or near the level of the LND (roughly 500 hPa). In this and the schematics presented in Figs. 3, 4, and 6, contours aloft represent lines of constant geopotential; contours at the surface represent lines of constant potential temperature. Filled arrows denote vertical motion

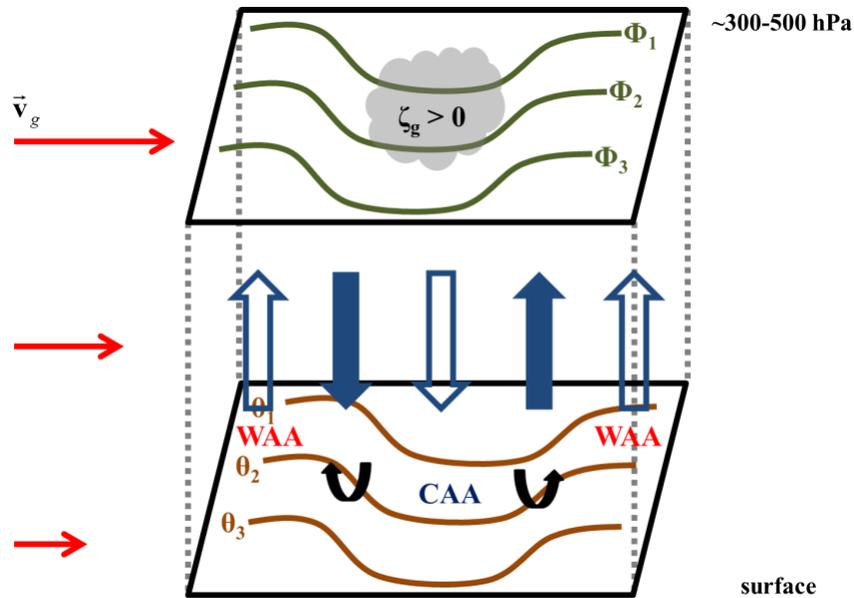
forcing due to geostrophic relative-vorticity advection at the LND; open arrows (present only in later figures) denote vertical motion forcing from thermal advection at the surface. WAA and CAA (present only in alter figures) denote warm and cold potential-temperature advection, respectively.

### *Step 2: Surface Development*

In Step 2, at some unspecified later time, the surface flow associated with the developing surface cyclone and anticyclone modifies the orientation of the lower-tropospheric isotherms from a near-zonal orientation to one with wave-like structure. This is indicative of cold potential-temperature advection between the surface anticyclone and cyclone and warm potential-temperature advection north and east of the surface cyclone. From our interpretation of (15), this causes *mid-tropospheric ascent and surface pressure falls northeast of the surface cyclone* and *mid-tropospheric descent and surface pressure rises southwest of the surface cyclone*. This results in the northeastward movement of the surface cyclone and southeastward movement of the surface anticyclone.

The aforementioned cold layer-mean potential-temperature advection is found beneath the upper-tropospheric trough. Likewise, the aforementioned warm potential-temperature advection is found beneath the upper-tropospheric ridge. From the quasi-geostrophic height-tendency equation, if the potential-temperature advections are maximized near the surface and decay upward, height falls occur in the base of the trough and height rises occur in the apex of the ridge, strengthening the upper-level pattern by amplifying the trough and ridge. This amplifies the geostrophic absolute vorticity (and its advection) associated with the trough-ridge pattern.

The continued – and enhanced – geostrophic absolute-vorticity advection at the LND works to further intensify the surface features via the same mechanisms noted in Step 1.



**Figure 3.** Idealized schematic of the upper- and lower-tropospheric pattern and accompanying forcings associated with the early development stage of a synoptic-scale surface cyclone.

The key take-home points to this point are as follows:

- The pattern of geostrophic absolute-vorticity advection with the upper-tropospheric features acts to intensify surface features.
- The pattern of thermal advection (and the accompanying vertical motions) associated with the surface features acts to intensify the upper-tropospheric features.
- A feedback loop thus exists as both sets of features move eastward.
  - The eastward motion of the surface features is driven by the thermal advection forcing described above.
  - The eastward motion of the upper-tropospheric features is driven by advection, as elucidated via our interpretation of the geostrophic absolute-vorticity advection term of the quasi-geostrophic height tendency equation.

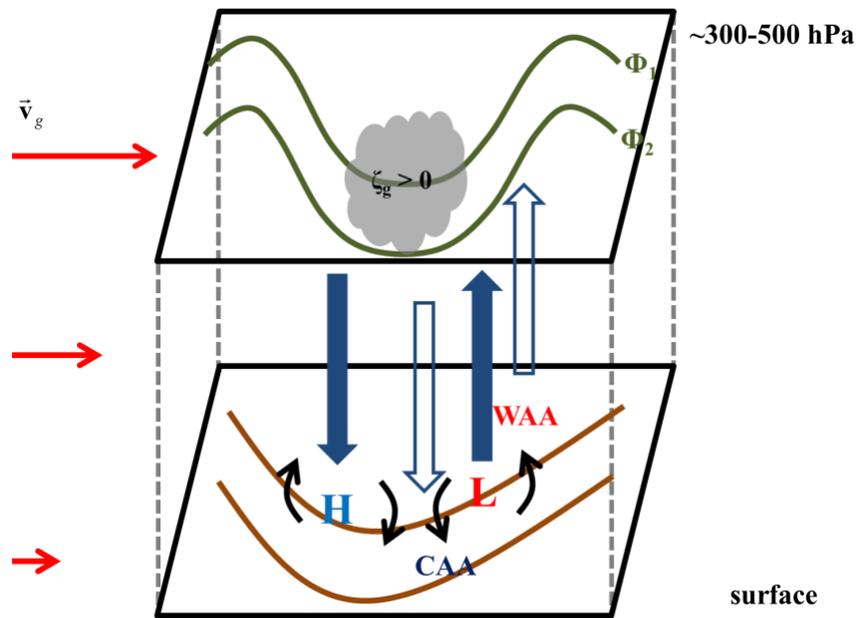
The fact that there is an upshear tilt, or one to the west with increasing height against the vertical wind shear, promotes this feedback loop. If this tilt remains constant, it is known as *phase locking*. You will likely hear more about phase locking in your study of waves in Dynamics II. If the tilt changes, the evolution of the pattern will change as the forcings evolve in both location and intensity. We will see this in action as we progress forward in time.

*Step 3: Development to Maturity*

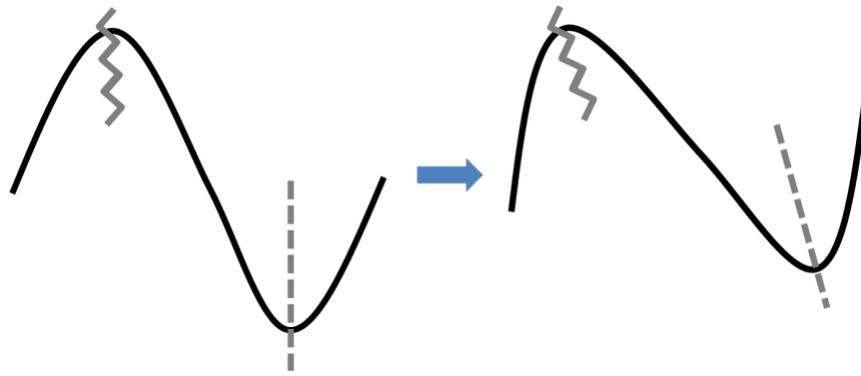
The intensification of the upper-tropospheric trough has intensified geostrophic absolute-vorticity advection aloft, thereby enhancing its associated patterns of ascent and descent and promoting the further intensification of the surface features. The intensified surface features continue to rotate the isotherms via advection, as discussed previously.

With warm potential-temperature advection maximized further northward with time, the surface cyclone takes on an increasingly large poleward (northward) component of motion. Likewise, with cold potential-temperature advection maximized further southward with time, the surface anticyclone takes on an increasingly large equatorward (southward) component of motion. While these surface features may appear to move in the same direction as the upper-tropospheric winds, this is coincidence – their motion is instead being driven by lower-tropospheric thermal advection.

Aloft, the reorientation of the lower-tropospheric thermal advection maxima leads to a concordant reorientation of where the forcing for middle-tropospheric height falls and rises is located: falls in the southern and eastern portion of the upper trough, rises in the northern and western portion of the upper ridge. This leads to both the trough and ridge becoming *negatively tilted* (i.e., from northwest to southeast in the horizontal plane). This is depicted in Fig. 5.



**Figure 4.** Idealized schematic of the upper- and lower- tropospheric pattern and accompanying forcings associated with the mature development stage of a synoptic-scale surface cyclone.



**Figure 5.** Idealized depiction of the transition from a neutrally tilted ridge/trough pattern early in a system’s lifecycle (at left) to one featuring a negatively tilted ridge/trough pattern as the system approaches maturity (at right).

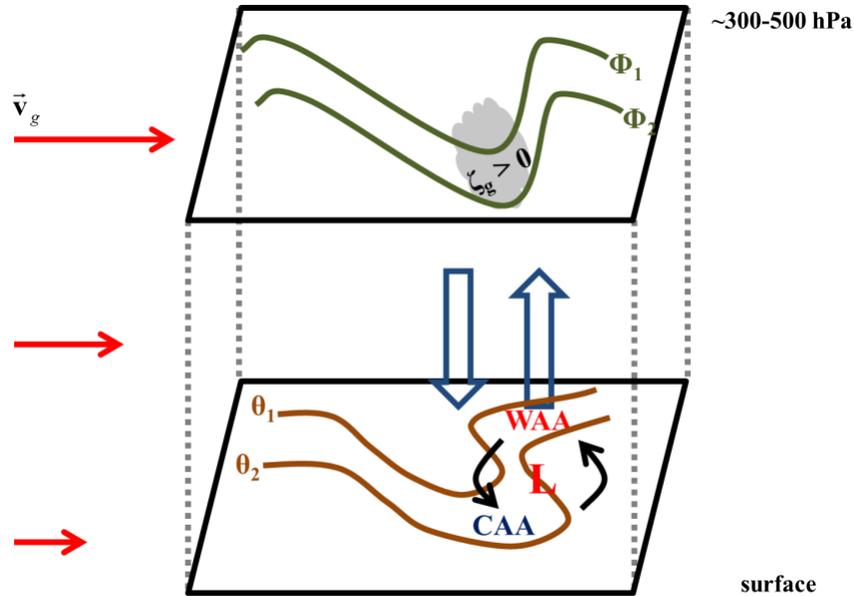
*Step 4: Maturity to Occlusion*

As the system matures toward occlusion (and its peak intensity), the lower tropospheric isotherm pattern becomes further distorted into an “S” shape, leading to warm thermal advection primarily north of the surface cyclone and cold thermal advection primarily south of the surface cyclone. This brings about even more of a northward motion of the surface cyclone.

The amplification of and increasing negative tilt to the upper-tropospheric ridge/trough pattern weakens the zonal component of the geostrophic absolute-vorticity advection aloft, slowing the eastward movement of the upper-tropospheric pattern. However, given the primarily poleward motion of the surface features, this reduces the tilt between the surface and upper-level features.

As a result, there is an increasingly large degree of overlap between the forcings for ascent and descent promoted by the lower- and upper-tropospheric features. This helps the surface cyclone to reach its maximum intensity. However, as the magnitude of the advectations gradually weakens both aloft and at the surface due to the amplification and tilt of the upper-level pattern and reorientation of the lower-tropospheric isotherms, respectively, the magnitude of these forcings weakens.

Eventually, the lower tropospheric thermal field becomes substantially distorted as it wraps around the cyclone such that there is little potential-temperature advection anywhere. Thus, the forcing upon the upper-tropospheric trough’s intensity and the surface cyclone’s movement goes away. Friction acts to spin down the surface cyclone (from (1)), while the upper-tropospheric trough may cut off from the synoptic-scale westerly flow as a result of its amplification. Without any forcing remaining to intensify it, it too will gradually decay unless or until some other forcing is imposed.



**Figure 6.** Idealized schematic of the upper- and lower-tropospheric pattern and accompanying forcings associated with the mature stage of a synoptic-scale surface cyclone.

Thus, the closed system given by the *quasi-geostrophic vorticity and omega equations*, as manifest through the *Pettersen-Sutcliffe development equation*, is sufficient to describe surface cyclone formation. When coupled with the *quasi-geostrophic height tendency equation*, the entirety of a synoptic-scale midlatitude surface cyclone and upper-tropospheric trough's coupled lifecycle can be described.